Turbulence in a Shock-Wave Boundary-Layer Interaction

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Direct measurements of the Reynolds normal and shear stresses have been made upstream and downstream of a two-dimensional, adiabatic, shock-wave boundary-layer interaction at $M=2.9$. The measurements were obtained independently with a laser velocimeter and a constant-temperature, hot-wire anemometer. In addition to these turbulence data, mean velocity data were obtained with the laser velocimeter and compared with data obtained from pitot-static probe. Obtained from the two independent systems, the turbulence data are compared with each other and with previous turbulence measurements. These data are examined in light of modeling the Reynolds stresses appearing in the streamwise momentum equation.

Nomenclature

a_w = overheat ratio for hot-wire anemometer, $(R_w - R_{\text{ref}})/R_{\text{ref}}$
d_f = fringe spacing
E = bridge voltage
ℓ = mixing length
M = Mach number
N = number in sample
p = pressure
R = wire resistance
Re = Reynolds number
T = temperature
u = velocity component in streamwise direction
V = velocity component normal to interference fringes
v = velocity component in cross-stream direction
x = streamwise coordinate, measured from tunnel reference station
y = cross-stream coordinate, measured from tunnel wall
$\Delta e_{()}$ = fluctuation sensitivity
δ = boundary-layer thickness
θ = included angle between beams
λ = wave length
μ = mean value in statistical discussion
ρ = density
σ = standard deviation
τ_{ij} = molecular stresses
τ_w = wall shear stress
$\langle\ \rangle$ = standard deviation (rms) of quality

Subscripts

o = conditions just upstream of interaction
t = total conditions
δ_o = upstream boundary-layer thickness
∞ = freestream conditions

Superscripts

$(\ )'$ = fluctuating quantity
$(\bar{\ })$ = time-averaged quantity
$(\tilde{\ })$ = mass-averaged velocity, $\tilde{u}=\overline{\rho u}/\bar{\rho}$

Introduction

IMPORTANT to the development of computing techniques for predicting the flow over aerodynamic configurations is the understanding of the turbulent transport mechanisms in the boundary layer. Currently, there is particular interest in the behavior of the turbulent boundary layer in regions of strong adverse pressure gradients caused by impinging shock waves. Numerous experimental studies of the shock wave boundary-layer interaction have been performed in the past.[1-3] However, with the exception of two studies,[4,5] the information obtained has been restricted to the mean-flow properties.

The mean-flow properties do not directly yield information about the behavior of the turbulent transport quantities such as Reynolds shear and normal stress, or the turbulent heat transfer. These latter quantities (or some mathematical model of them) are required in computing the behavior of flows governed by the complete conservation equations. Mean-flow measurements can be difficult to make in the vicinity of shock waves and, even if the mean flow properties were known exactly, indirect techniques[6] for "backing out" turbulent transport information cannot yield all of the components of the Reynolds stress tensor. Thus, direct measurements of the turbulence properties are sought.

Only one previous study[5] has produced direct measurements of the Reynolds normal and shear stresses in a shock-wave boundary-layer interaction. A hot-wire anemometer was used in that study to obtain the turbulence information. Since the hot-wire anemometer is sensitive to pressure fluctuations and a shock-wave boundary-layer interaction produces substantial pressure fluctuations,[4] the validity of the reported measurements (which do not include the pressure effects) in the immediate location of the shock waves may be open to question. This might be resolved by using the laser velocimeter, a measurement technique which is sensitive to velocity fluctuations only. In addition to possibly shedding light on the pressure fluctuation question, the laser velocimeter and hot-wire anemometer can be used in conjunction to support the validity of the turbulence information provided by the two independent measurement systems.

In the present investigation, the constant-temperature hot-wire anemometer and laser velocimeter were used to obtain information on the interaction of an oblique shock wave with a turbulent boundary layer. Mean-velocity profiles were obtained with the laser velocimeter and are compared with those obtained with pitot and static pressure probes in a previous study of the same flow.

Turbulence measurements of streamwise and cross-stream velocity fluctuations $\langle u'\rangle$ and $\langle v'\rangle$ and their correlation $\overline{u'v'}$ were made with both the hot-wire and laser systems. These measurements were used to investigate (for the present flow conditions) the magnitude of each term which appears on the right-hand of the streamwise momentum equation that is usually used to describe two-dimensional, turbulent flow:[7]

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$$\partial\bar{\rho}\bar{u}^2/\partial x+\partial\bar{\rho}\bar{u}\bar{v}/\partial y=-\partial\bar{p}/\partial x$$
$$+\partial/\partial x(\tau_{xx}-\bar{\rho}\overline{u'^2})+(\partial/\partial y)\ (\tau_{xy}-\bar{\rho}\overline{u'v'}) \qquad (1)$$

The magnitude of the streamwise derivative of Reynolds' normal stress $\partial(-\bar{\rho}\overline{u'^2})/\partial x$, usually neglected in boundary-layer analyses, is compared with $\partial\bar{p}/\partial x$ and with $\partial(-\bar{\rho}\overline{u'v'})/\partial y$, the cross-stream derivative of the Reynolds' shear stress throughout the flow. Also, the mathematical modeling of the Reynolds stresses that are required to describe their observed behavior, is discussed.

Apparatus and Measurement Techniques

The experimental arrangement used in this study is shown schematically in Fig. 1. The turbulent boundary layer developed on the adiabatic wall of a planar wind-tunnel nozzle and test section. The test section is 20.3 by 20.3 cm with conditions upstream of the incident shock wave of $M_\infty=2.9$ and $Re_{\delta_o}=1.4\times10^6$. The incident shock wave was produced by a full-span generator set at a 7° angle of incidence to the freestream flow, producing approximately a 2.5 to 1 pressure rise across the interaction. The mean-flow properties for this interaction were obtained in another study[8] by using pitot and static tubes and wall-pressure orifices. The mean-flow information indicates that this particular interaction did not separate the boundary layer. The following sections briefly describe the laser velocimeter and hot-wire anemometer techniques employed in the present study.

Laser Velocimeter

The difficulty of obtaining compressible flow measurements by laser velocimetry are indicated by the few measurements reported in the literature. The majority of these have been limited to mean velocity determinations. Several investigations[9-11] have reported the use of laser velocimetry to obtain the measurement of the streamwise velocity fluctuations $\langle u'\rangle$. The measurements of the cross-stream velocity fluctuations $\langle v'\rangle$ and the correlation $u'v'$ are limited to those presented in Refs. 11 and 12.

The laser velocimeter in the present study was a one-component, forward-scatter system with the standard dual-scatter or fringe-mode optical configuration. Figure 2 depicts the laser velocimeter and its orientation to the wind-tunnel test section.

The velocity component sensed with this optical arrangement lies in the plane of the two incident laser beams and is perpendicular to their bisector. Thus, various degrees of sensitivity to the u and v velocity components, with the maintenance of an insensitivity to the other orthogonal component, were achieved by rotating the splitter cube about an axis that was coincident with the incoming laser beam. The signals obtained at various splitter-cube orientations were processed [11] to yield $\langle u'\rangle^2$, $\langle v'\rangle^2$, and $\langle u'v'\rangle$.

Figure 3 is a block diagram of the signal processing arrangement. The single-particle, signal processor was essentially the same as the signal processor described in Ref. 14

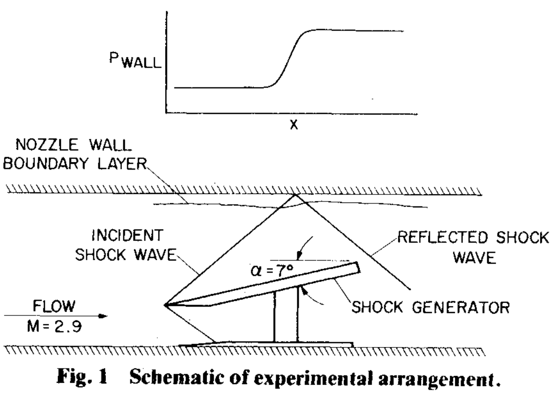

Fig. 1 Schematic of experimental arrangement.

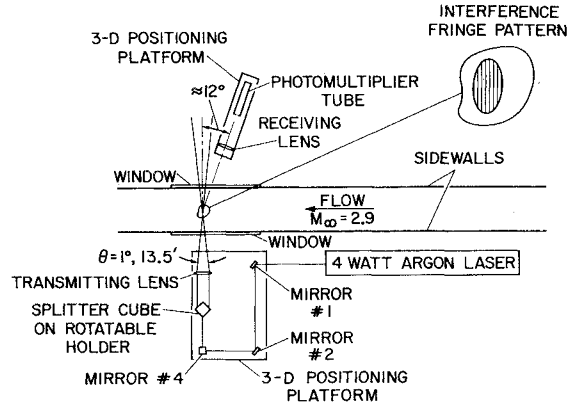

Fig. 2 Schematic of laser velocimeter.

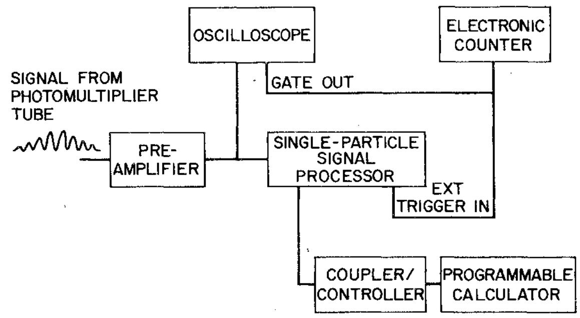

Fig. 3 Schematic representation of signal processing arrangement.

The calculator was programed to calculate the instantaneous velocity which corresponds to each time interval that was provided by the signal processor and also to calculate the mean value and standard deviation of these instantaneous velocity readings from the following statistics:

$$\bar{V}=\sum_{i=1}^{N}\frac{V_i}{N}\ ;\ \sigma=\langle V'\rangle=\left[\sum_{=1}^{N}\frac{V_i^2}{N}-\bar{V}^2\right]^{1/2}$$

As indicated by the relationships above, no adjustments were made for the Doppler ambiguity, since this is not appropriate in single-particle, time domain measurements.[15] Obtained in the data reduction were several samples, each of which contained 200 or 400 instantaneous velocity measurements.

Only the naturally occurring particulates in the flow were used for light scattering. In this sense, the flow was found to be exceptionally clean, possibly because the air supply was dried to a specific humidity of less than 0.0001. The maximum number of particles that were detected crossing the sensing volume was $\simeq 200\ \text{sec}^{-1}$. To obtain an estimate of the size of the naturally occurring particles, tobacco smoke (diam <1 μm) was injected into the flow and the scattered light levels compared with those produced by the naturally occurring particles. Since light levels observed were nearly the same, the particle sizes must have been similar.

More important than the actual particle size is the ability of the particle to respond to turbulent fluctuations or discontinuities in velocity across shock waves. The relaxation distance of the naturally occurring particles as they passed through the reflected shock wave shown in Fig. 1 was determined by making a streamwise scan outside the boundary layer ($y=2.3$ cm) of the cross-stream velocity component. The cross-stream velocity changes markedly across the shock wave because of the 7° turn in the flow. This characteristic of the cross-stream velocity component makes it ideal for determining the relaxation distance, which was approximately 0.4 cm. The measurement supports the estimate for a particle size of 1 μm and corresponds to a frequency response of 24 kHz in the moving frame of reference of the particle. In a fixed frame of reference, or Eulerian reference frame, for which frequency responses are usually given, this frequency of 24 kHz would correspond to a frequency several fold greater, because of the large convection velocities present in supersonic flows.

From oscilloscope displays of the single-particle signals, the laser beam diameter at the beam crossover point, which determines the effective spatial resolution of the laser velocimeter, was estimated to be 300 μm (based on the $1/e^2$ intensity point).

To minimize the effects of spurious measurements that could result from the infrequent, but not negligible, noisy signals, the velocity readings were tested on the mean value and standard deviation computed on the previous sample ($N=200$ or 400). Measurements not satisfying the condition

$$|V_i - \bar{V}| \leq 3\sigma$$

were rejected by the calculator in the compilation of the next sample. This 3σ test was selected since, for probability density functions typical of those encountered in the present study, the error in σ created by imposing this condition was small compared with the statistical uncertainty created by the finite sample size used.

With the 3σ condition applied in the presence of spurious measurements, the calculated σ would first decrease with subsequent samples and then oscillate about a nominal value as a result of statistical uncertainty (finite sample sizes). The samples accepted as valid data were those following and including the sample that preceded the first observed increase in the standard deviation. The effect of spurious measurements were most noticeable in regions of low turbulence—the outer portion of the boundary layer. In the inner two-thirds of the boundary layer, little or no difference in σ resulted from applying the 3σ test. All of the laser velocimeter measurements presented are based on several thousand instantaneous velocity determinations at each measurement station and laser beam orientation.

Hot-Wire Anemometer

The use of hot-wire anemometers for turbulence measurements in supersonic flows is, of course, not new. The general theory of operation and signal interpretation was outlined nearly 25 years ago.[16-18] Since then, relatively few works have appeared in the literature, because of the increased problems with hot-wire experimentation in supersonic flows over subsonic flows. Wire breakage, extraneous electrical signals caused from the mechanical strain of the wire, and calibration are some important problems to be overcome in supersonic flows.

Beyond the calibration and mechanical problems of hot wires, the process of signal interpretation used in resolving the fluctuations of the physical variables (density, velocity, total temperature, pressure, etc.) from the measured voltage fluctuations is considerably more complex in supersonic flows.

Several authors[19-22] have reported measurements of streamwise velocity fluctuations $\langle u' \rangle$ that were obtained by using a normal-wire probe; i.e., one in which the sensing wire is held at a right angle to the streamwise direction. However, to obtain these velocity fluctuations, the signal interpretation procedure requires an assumption regarding the near-field pressure fluctuation p'. The assumption in all cases of supersonic flow has been simply that $p'=0$. However, in one case[23] of hypersonic flow, the pressure fluctuations were estimated by making certain assumptions concerning the correlation of the pressure fluctuations with those of entropy and vorticity. In the present work, the $p'=0$ assumption was indirectly investigated by comparing the laser velocimeter measurements (which do not rely on any p' assumption) with those of the hot-wire anemometer.

In contrast to the number of investigators reporting $\langle u' \rangle$ information, only three works[5,11,22] have reported measurements of the cross-stream velocity fluctuation $\langle v' \rangle$. This measurement requires the use of wires that are yawed to the streamwise direction. In Ref. 22, the measurements were obtained at a single location in a supersonic boundary layer by using the instantaneous difference of two wires from a cross-wire probe (the "X-meter," a probe that consists of two yawed wires at $\pm 45°$). Extraneous strain-gage signals and wire-matching problems affected the value of $\langle v' \rangle$ obtained in that study. In Ref. 5, an entire set of data was obtained by using a single yawed wire, which could be rolled to the plus and minus positions, to eliminate the wire matching problem; but other factors which may have affected the accuracy of that technique were involved in the determination of $\langle v' \rangle$. Measurements of $\langle v' \rangle$ were made in Ref. 11 and in the present study by an X-meter and are apparently free of strain-gage signals and matching problems.

In addition to the individual components of the velocity fluctuation $\langle u' \rangle$ and $\langle v' \rangle$ (to obtain the Reynolds normal stresses), it is desirable to know the correlation between u' and v', i.e., $\overline{u'v'}$ (to obtain the Reynolds shear stress). Measurements of $\overline{u'v'}$, again by using yawed wires, in supersonic flows are also limited to the three previously cited investigations.[5,11,22] Accuracy of the data presented in Refs. 5 and 22 has not yet been ascertained; however, in Ref. 11, results of measurements made by using the techniques of Refs. 5 and 22 compared favorably with the results obtained by using the laser velocimeter for a zero pressure gradient flow. This comparison is made for the flow downstream of a strong adverse pressure gradient in the present study.

The wires used in this investigation were 5 μm tungsten, spot welded to electrodes inbedded in epoxy. The length of the normal wire was about 0.8 mm and the lengths of the yawed wires were about 1.2-1.3 mm. The five over-heat ratios used in this study were $a_w=0.2$, 0.3, 0.4, 0.6, and 0.8 for both the normal and yawed wires. The data were reduced as described in Ref. 11 using the $p'=0$ assumption. The cross-stream velocity fluctuation $\langle v' \rangle$ was found directly,[11] without assuming $p'=0$. Closely-matched, yawed wires (at approximately $\pm 45°$ to the flow direction) were instantaneously differenced and time averaged to obtain $\langle v' \rangle$. The wires were considered matched when their visual appearances were similar, their cold (reference) resistances were within 10% of each other, and their sensitivity coefficients were within less than 5% of each other over the range of wire temperatures used ($0.2 \leq a_w \leq 0.8$). Considerable time was required to achieve this matching, however, it is felt that this is a more reliable technique than those used in some previous studies in which the determination of $\langle v' \rangle$ required the differencing of two large numbers.

Results and Discussion

The results of the present investigation comprise: mean velocity profiles obtained with the laser velocimeter; the turbulence quantities (i.e., $\langle u' \rangle$, $\langle v' \rangle$, etc.) obtained with the laser and hot-wire anemometers; and inferences made for modeling the terms shown on the right-hand side of Eq. (1). Figure 4 shows a plot of the surface static pressure distribution aligned with a scale drawing of the incident and reflected shock-wave system. As can be seen, the measuring stations, one upstream ($x=13.7$) and two downstream ($x=21.3$ and 23.8), are in regions where the local streamwise pressure gradient is about zero, i.e., $\partial\bar{p}/\partial x \simeq 0$.

Mean velocity $\bar{u}$ profiles that were obtained with the laser anemometer are compared in Fig. 5 with those deduced from pitot-static probe data.[8] The excellent agreement between the compared profiles lends strong support to the validity of using the laser velocimeter for determining mean velocities in regions of large pressure gradients and shock waves.

The time-averaged streamwise and cross-stream velocity fluctuations $\langle u' \rangle$ and $\langle v' \rangle$ obtained with the laser and hot-wire systems are shown in Fig. 6 for the upstream and both downstream stations. The very good agreement between the two independent systems for obtaining $\langle u' \rangle$ is encouraging

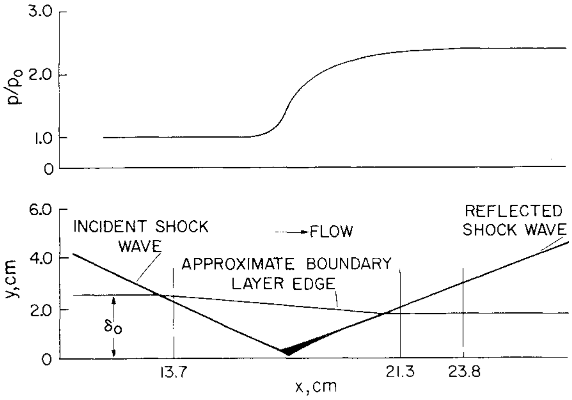

Fig. 4 Flowfield and surface pressure distributions showing measurement stations.

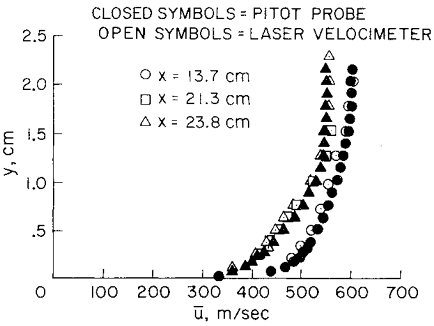

Fig. 5 Mean velocity profiles.

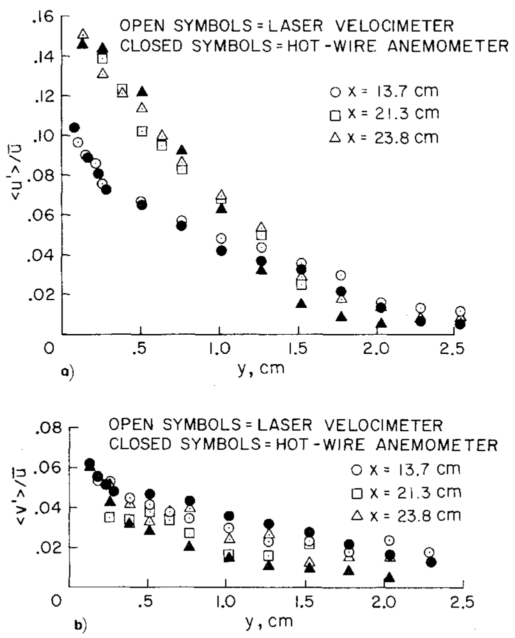

Fig. 6 Velocity fluctuations: a) streamwise fluctuations; b) cross-stream fluctuations.

and lends credence to the fluctuation measurements obtained by both systems. The magnitude of $\langle u' \rangle$ is increased substantially as a result of the interaction. Regarding the assumption that $p' = 0$, which was used for reducing the hot-wire data, we can conclude from the agreement between results obtained by using the two systems that, if pressure fluctuations are present, they have little effect on the final values of $\langle u' \rangle$, since differences between the compared results are probably within the uncertainty of the two techniques. The measurements of $\langle v' \rangle$ do not indicate the clear change from upstream to downstream seen in $\langle u' \rangle$. Although there is general agreement between the laser and hot-wire systems for $\langle v' \rangle$ upstream of the interaction, the effect of the interaction on $\langle v' \rangle$ is not clear. The laser indicates essentially no change in the magnitude of $\langle v' \rangle$ at comparable values of y while the hot-wire shows a decrease. Differences in results from the two measurement systems may be due to inherent difficulties of both systems. For example, the wires might not be exactly matched or the laser data processing technique might introduce inaccuracies when $\langle v' \rangle^2$ is much smaller than $\langle u' \rangle^2$. In spite of these or other possible sources of uncertainty, the measurements do show that no large increase in $\langle v' \rangle$ (similar to that observed for $\langle u' \rangle$) occurs as a result of the interaction.

Using the values of $\langle u' \rangle$, we may obtain the Reynolds normal stress $\bar{\rho}\overline{u'^2}$ [refer to Eq. (1)], as shown in Fig. 7. Results obtained by the laser and hot-wire systems for Reynolds shear stress $\bar{\rho}\overline{u'v'}$ used in Eq. (1) are shown in Fig. 8, along with the expected wall values of the total shear stress, which were obtained by fitting the mean velocity data to a law-of-the-wall profile. The Reynolds' shear stress measurements upstream of the interaction show an unexpected decrease in magnitude near the wall rather than continuing to the wall value from the level measured at about $y = 0.7$ cm. Reasons that could account for this decrease, indicated by both measurement systems, have not been found as of yet. The measurements at the downstream station, however, do indicate a trend toward the wall value.

What then are the implications relative to modeling the Reynolds stress terms shown in Fig. 7 and 8? The gradients of the Reynolds stresses required in Eq. (1) were computed from the smoothed data of Figs. 7 and 8. These gradients, i.e.,

$$\partial\bar{\rho}\overline{u'^2}/\partial v \quad \text{and} \quad \partial\bar{\rho}\overline{u'v'}/\partial y$$

are shown in Fig. 9 at the farthest downstream station (for which $\partial\bar{p}/\partial x \simeq 0$). That the normal stress gradient is indeed much smaller on the average than the shear stress gradient concurs with usual boundary layer analyses. Thus it is clear,

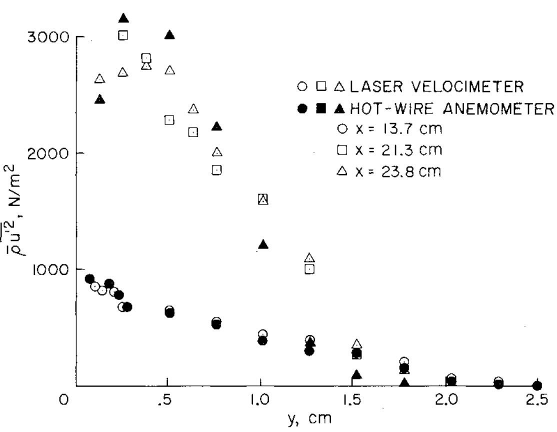

Fig. 7 Reynolds normal stress.

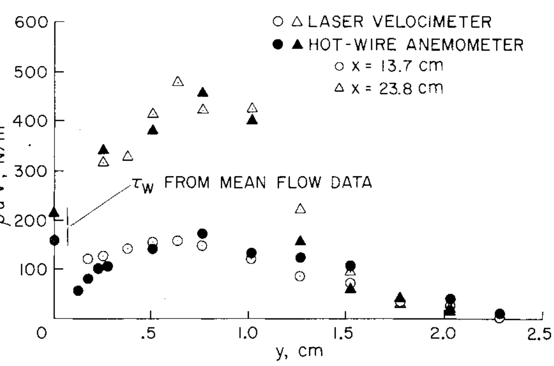

Fig. 8 Reynolds shear stress.

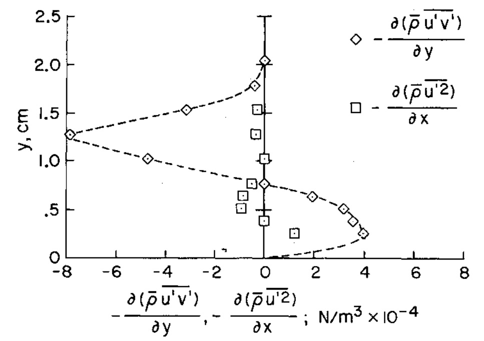

Fig. 9 Gradients of Reynolds stresses downstream of interaction.

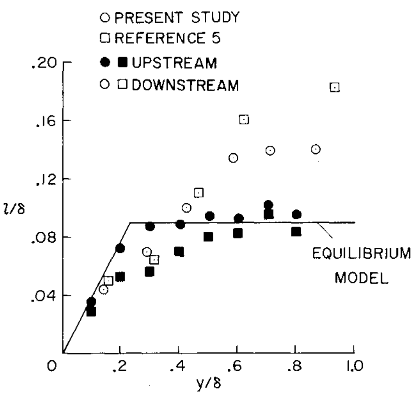

Fig. 10 Dimensionless mixing length distributions.

for this case at least, that the only term that need be modeled in the downstream boundary layer is simply

$$\partial\bar{\rho}\overline{u'v'}/\partial y$$

Whether or not $\partial\bar{\rho}\overline{u'^2}/\partial x$ can be neglected throughout the interaction can be discussed with the aid of Fig. 7. It is quite clear that $\partial\bar{\rho}\overline{u'^2}/\partial x$ is certainly not small between the upstream and downstream stations. In this region, however, we must compare the normal stress gradient with the large pressure gradient term $\partial\bar{p}/\partial x$. Although no turbulence measurements were made in the pressure gradient region, we may compare the normal stress gradient (see Fig. 7), which for order of magnitude estimates, is assumed to be constant between the upstream and downstream stations with the average pressure gradient. This comparison indicates that the value of the average pressure gradient is near 5×10^5 N/m³ while the maximum normal stress gradient is about 4×10^4 N/m³. Assuming that $\bar{\rho}\overline{u'v'}$ increases monotonically from theupstream to downstream station, it is evident, from Figs. 8 and 9, that the term $\partial\bar{\rho}\overline{u'v'}/\partial y$ is everywhere less than about 8×10^4 N/m³ in this region. Therefore, for this particular flow, we may reasonably conclude that, within engineering accuracy, the term $\partial\bar{\rho}\overline{u'^2}/\partial x$ may be neglected throughout the entire flow upstream, within, and downstream of the shock-wave interaction region. We may further conclude from the presented data that, within the interaction itself, the pressure gradient term $\partial\bar{p}/\partial x$ is an order of magnitude larger than the Reynolds stress gradients.

We are now left with only the Reynolds shear stress, which must be modeled in the downstream flow. A concept that has been used successfully to model the Reynolds shear stress in equilibrium and near-equilibrium boundary-layer flows is that of a mixing length. Various formulations of this quantity have been proposed, but one of the more common ones is shown for reference in Fig. 10 along with the data obtained from both the present study and that of Ref. 5. The flow described in Ref. 5 is a turbulent boundary-layer shock-wave interaction that occurred in an axially symmetric facility. The Mach number and Reynolds numbers were about $M_\infty = 4$ and

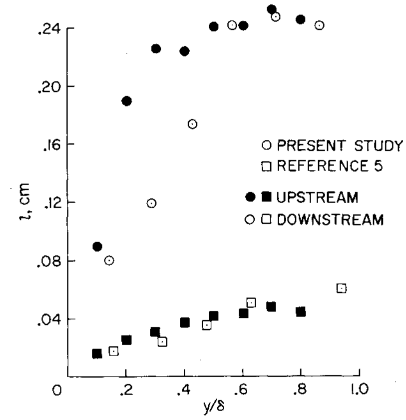

Fig. 11 Mixing length distributions.

$Re_{\delta_0} = 9\times10^4$. The pressure rise across the interaction was about 3 to 1; and, as in the present study, the flow was not separated. The values of ℓ were calculated from the usual relationship

$$\ell = (\bar{\rho}\overline{u'v'}/\bar{\rho})^{1/2}/(\partial\bar{u}/\partial y)$$

From the data presented in Fig. 10, it is clear that the equilibrium mixing length model does not account for all the observed variations of ℓ/δ. However, near the wall, say $y/\delta <$ 0.1, it does appear that all the data are in reasonable agreement with the equilibrium curve whose slope is 0.4 (the von Karman constant). In addition, it can be concluded that the upstream boundary layers (both nozzle wall flows) are near equilibrium, since the values of ℓ/δ away from the wall tend to the equilibrium value of about 0.09. But since the downstream shear stresses are far from equilibrium, another approach to their modeling must be sought.

A concept that is opposite to that of equilibrium turbulence is "frozen" turbulence, i.e., a situation wherein the turbulence downstream of a sudden change is the same as that upstream of the change. To assess whether or not the shear stress away from the wall might be better described by a frozen turbulence model, we need only examine the values of the mixing length ℓ, as shown in Fig. 11. These values were obtained upstream and downstream of the interaction described in the present study and that of Ref. 5. Beyond y/δ of about 0.4, the values of ℓ are widely different for the two different flows. It therefore appears that the simplest model for Reynolds shear stress that is consistent with the observations of the only two studies in which direct shear stress measurements in pressure gradient flows have been made is the following: Near the wall, the usual equilibrium model appears valid; away from the wall, the mixing length may be considered for practical purposes to be frozen, and ℓ just downstream of the interaction to be set equal to ℓ upstream. In the downstream flow, ℓ must eventually attain its equilibrium value in the absence of further severe pressure gradients. A reasonable assumption, consistent with incompressible flow behavior,[24] is simply that there is an exponential relaxation to equilibrium. The relaxation length consistent with compressible[25] and incompressible[24] flow observations is roughly between 5 and 15 boundary-layer thicknesses. If $10\delta_o$ is chosen as the length, an expression that might be used to describe the behavior of ℓ away from the wall region is

$$\ell_{\text{downstream}} = \ell_{\text{upstream}} + (\ell_{\text{downstream, equilibrium}} - \ell_{\text{upstream}})$$
$$\{1 - \exp - [(x - x_o)/10\delta_o]\}$$

Conclusions

Measurements of mean velocity profiles, fluctuating velocities, and Reynolds stresses have been made upstream and downstream of an unseparated, boundary-layer shock-wave interaction. Having examined these measurements with respect to modeling the Reynolds stresses that appear in the streamwise momentum equation, we can conclude that 1) within engineering accuracy, the Reynolds normal stress may be neglected throughout the flow, 2) the gradient of Reynolds shear stress was substantially less than the pressure gradient within the immediate interaction region, and 3) the Reynolds shear stress downstream of the interaction may be described by a combination of an equilibrium and frozen mixing-length model.

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